Decision Making in a Multi-Branch Bank and Information Aggregation

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Decision making problem of a bank consisting large number of branches and the head office that supplies «collectively used inputs» is considered.

1. Introduction

A bank can be considered as an organization with a head office (representing the bank management, responsible for guiding the bank towards its objectives approved by its board of directors) and branches (responsible for producing & selling banking services to achieve targets assigned to them). In guiding bank, the head office’s main task is to make sure that bank branches use their full capacities efficiently in line with the directions given. Such a task, despite its apparent simplicity is, in fact, a difficult one as is evidenced by the richness of results that the principal-agent theory approach offers. In the principal-agent theory the problem is formulated as follows: The principal (head office in this case), is the position of making decisions on the basis of information provided by the agents (branches) and/or allow these agents to make decisions on her behalf. Agents, on the other hand, have their own preferences which may or may not be full compatible with the principal’s aims. Therefore, in the case of a bank, it is the head office’s responsibility to ensure that branches act in bank’s best interest instead of their own. In undertaking such a task, the head office faces two types of informational problems. First, it is extremely difficult, if not impossible, for the head office to observe all the relevant actions of branches.

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2) For a lucid discussion of the principal-agent theory see [12, p. 477–510].
(hidden action). That prevents the head office to fully grasp branches’ ability in fulfilling their obligations. Secondly branches, in general, are better endowed with information related to their environment and their capabilities (hidden information).

Attempting to overcome these informational asymmetry problems by devising a mechanism to transfer all information from the branches to the head office for processing will be prohibitively costly, and therefore infeasible. A much more rewarding approach is, then, to start with imperfect information assumption and reduce its negative effects by information selection (i.e. introducing an ordering on the information set to identify the most relevant ones) and (or) by information aggregation. In practice, information selection is considered as one of the major responsibilities of the management and therefore it is a widely discussed topic. Information aggregation, on the other hand, is a rather neglected issue since in most instances it is presumed to lead information loss. Therefore information aggregation is either ignored or when it is implemented, except in the well defined world of accounting, it is done without paying sufficient attention to its rather delicate methodological problems.

It is clear that banks are not the only organizations that face such informational problems. There is a growing body of research that takes firm as consisting of separate agents, with different motives. The operation of the firm requires a guided coordination of their efforts, which is achieved through its internal organization. The organization itself on the other hand, can be defined by the pattern of information exchange among such agents, DeCanio & Watkins [5, p. 278].

One distinguishing feature of banks, in this context, is their reliance on information. All bank services are produced with technologies that can be characterized as «information intensive». Therefore, informational flows are not only necessary for managing the banks, but also vital in performing its activities at the branch level. This point is particularly important, when one takes into account the cost of information processing. Banks, in order to operate, need to collect/process information concerning the environment they are operating. Some of this information is local, therefore collected by branches and partly processed by them. However a great bulk of information is either collected or processed at the head office. Therefore, in formalizing a bank, the head office should be treated not only as responsible from guiding the branches in line with banks objectives, but also as a unit that supplies information, i.e. a collectively used input.

In this paper, decision making problem of a bank consisting large number of branches and the head office that supplies «collectively used inputs» is considered. It is further assumed that the head office can collect reasonably reliable accounting and statistical information from branches concerning their banking operations. In the second section of the paper such a bank is described with the help of the terminology borrowed from the theory of resource allocation mechanisms. The third section is devoted to the problem of maximizing profit for the bank in question is discussed by considering centralized and quasi-decentralized solutions. It is claimed that, the first approach is informationally infeasible and the second is inefficient. In

3) See [8], for a selective analysis of the major theories of the firm. For the views of the institutional school that places a strong emphasis on the concept of firm as an organization see [4, 7].
4) See [1] on resource allocation mechanisms.
the fourth section, the problem of information aggregation as a means to deal with imperfect information problem is addressed. In this context, a particular information aggregation scheme, proposed in [6], to cluster bank branches according to their environmental characteristics is discussed. In concluding remarks, some practical side benefits of conducting such a study are pointed out. The algorithm used in constructing clusters is given in the Appendix.

2. Modelling Bank as an Interdependent System of Units

Consider a bank with a head office (denoted by 0) and n branches, where n is large. Let \( i \) denote the \( i^{th} \) branch of the bank and \( i \in \Omega = (1,\ldots,n) \). The bank in question is assumed to offer a bundle of banking products (credit, deposit, CD, LC etc.). It is assumed that all these products are produced by branches. The head office, on the other hand, is assumed to be responsible from the production of services that are collectively used as inputs by branches, such as supplying information concerning the economic environment that the bank is operating in, offering legal services for the bank as a whole and devising/enforcing internal rules and regulations etc. In other words, the head office can be considered as supplying a set of services to be used collectively by bank branches as inputs.

Let
\[
y = [y_j]_{\Omega} \quad j \in J,
\]
denote the vector of banking products and
\[
y_j = \sum_{y \in \Omega} y_j,
\]
where \( y_{ij} \) : banking product \( j \) associated with bank branch \( i \). If \( y_{ij} > (<) 0 \), then branch \( i \) is a producer (user as input) of banking product \( j \).

\[
\bar{z} = [z_i]_{\Omega}
\]
be the vector of collectively used inputs that the head office is offering.

The units of the bank communicate among each other through exchanging messages. For example, the head office sends targets for a specified period to branches and in turn branches respond by sending periodic information concerning their performances. Let
\[
\mu_i = \left( \mu_{ij} \right)_{i \in \Phi} \quad i \in \Phi
\]
be the index set for all units of the bank is defined as \( \Phi = \emptyset \cup \Omega \).

In this framework, a bank branch that collects more deposits than it needs to finance the loans it extended is assumed to lend it to other bank branches; i.e. an inter-branch market for funds exist. On the other hand, for the sake of simplicity, when bank borrows from other financial institutions it is assumed to be accounted as a deposit in one of its branches, despite the fact that such operations are carried out by the head office.
be the message vector sent by the $i^{th}$ bank unit (including the head office) at time period $t^7$. Such a message is a response to the messages received by the $i^{th}$ unit from others and therefore can be considered as a function of them and the specific information concerning the environment it operates, ($\Lambda_i \subset \mathfrak{I}$, where $\mathfrak{I}$ is the environment that the bank is operating in).

A bank unit’s message correspondence depends on the messages it received from other units and its knowledge about the environment that the bank is operating in. For the sake of simplicity assume that bank units have short memory, i.e. they can only use the messages they received in the previous period from other units and they can only have information their part of the environment. Then

$$\mu_i^t = \eta \left( \left\{ \mu_{i^t} \right\}_{i \in \Phi, \Lambda_i} \right), \quad i \in \Phi$$

Each bank unit is assumed to be making its own decisions and implementing them after reaching an understanding with other units. In this context, understanding can be formalized as a state in which the units repeat the same messages; i.e. if

$$\mu_i^t = \mu_i^{t-1} = \mu_i^\ast \text{ for } \forall i \in \Phi \quad \& \quad \text{for some } t \geq 1$$

then $\mu_i^\ast$ is called the equilibrium message of $i$, and

$$\mu_i^\ast = \left( \mu_1^\ast, \ldots, \mu_n^\ast \right)$$

is the vector of equilibrium messages$^8$.

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$^7$ The choice of the length of time period depends on the nature of the problem at hand. The speed of working of financial markets, as well as their volatility, requires (6) to be defined in the high frequency domain. For example, branches need to get information concerning exchange rate and stock exchange movements almost continuously. On the other hand, the most frequent macroeconomic information that is released to public is at monthly basis (e.g. inflation figures). In practice, decision making bodies of a bank communicate with each other almost continuously. However, transmission of relevant information that affects the behavior of decision making bodies require some administrative preparations and therefore their frequencies are much lower. For example, in most banks, asset-liability management committees meet once a week and transmit its evaluation of the developments and expectations concerning financial markets to the branches. A general overview of the bank’s performance in the light of the macroeconomic developments and financial market trends requires longer periods. In general, banks undertake such evaluations on quarterly basis, coinciding with their legal obligations of revealing their financial statements to the public.

$^8$ Most of the messages exchanged among bank units transmit information expressed in quantitative terms: such as interest rates, exchange rates, quantitative targets and credit scores for customers. Under such circumstances, as in most economic models Euclidean message space serves the purpose. For a recent survey of the alternative approaches to communication procedures and the characteristics of information spaces, see Van Zandt [16, section 5].
Suppose that bank units base their actions on equilibrium messages, i.e. they base their decisions upon the agreement among them\(^9\). Then the outcome function for the head office can be expressed as

\[
\langle z_x, x_0 \rangle = \varphi_0 \left[ \left\{ \mu_{z_0} \right\}_{z \in \Omega} \cdot \Lambda_{x_0} \right],
\]

where \(x_0\) is the vector of non-financial inputs of production (such as different types of labor and capital etc) and \(\hat{z}\) is the output (collective goods) vector. \(\Lambda_{x_0}\) is the environmental characteristics vector for the head office (such as its organization, capabilities etc.) On the other hand the outcome function of a branch can be written as

\[
\langle y_i, -x_0, -\hat{z} \rangle = \varphi_i \left[ \left\{ \mu_{z_i} \right\}_{z \in \Phi} \cdot \Lambda_i \right], \quad i \in \Omega.
\]

In contrast to head office bank branches are assumed to use both the collective goods that the head office produces and non-financial inputs of production, \(x_i\). As was indicated above, some of the elements of \(y_i\) vector may be negative, i.e. one branch may use one banking product as an input. Notice that in this formulation, obtaining financial inputs also requires an effort on behalf of the branch. For example, a bank branch that collects deposits and extends loans has to allocate its own resources to both activities.

3. Bank’s Profit Maximization Problem: “Centralized” and “Quasi Decentralized” Solutions

A bank with many may either choose to devise a strategy and implement it through centralizing its decisions or developing a more decentralized structure. The purpose of this section is to draw attention to the well-known problems of centralized decision making and also the limits of decentralization.

Consider the following “centralized” formulation of bank’s profit maximization problem:

\[
\max \Pi = \sum_{i \in \Omega} p_{y_i} y_i - \sum_{j \in \Phi} w_{x_j} x_j
\]

subject to

\[
\gamma_0 \left( z_x, x_0 \right) \leq 0
\]

\(^9\) This is obviously a simplifying assumption in the spirit of the Walrasian tatonnement procedure, which implies a deterministic communication framework, as shown in [13]. On the other hand, as was pointed out in [9, p. 302], informational imperfections can be formalized by introducing variables that reflect errors in perception.
where \((\tilde{p}_i)\) is the vector of the local (i.e. branch specific) prices of banking products and \((w_i)\) is the vector of the local prices of non-financial inputs used by the bank branch \(i\). \(x = [x_{ij}]_{n \times 1}\). Here \(\gamma_0\) and \(\gamma_j, j \in \Omega\) represent the production technologies of the head office and branches, respectively. Notice that in (10a) costs of producing services supplied for collective use of bank branches are also taken into account.

The first constraint of the problem, (10c), refers to production possibilities for branches. (10d) refers to the fact that each branch has to use some non-financial input for producing banking products. (10e) refers to aggregate input constraint of the bank. Bank has limited endowments of non-financial inputs that are demanded by its branches as well as its head office. It is assumed that this amount is given, but the management can reallocate them among bank units. Finally, (10f) implies that the services supplied by the head office are demanded.

Let’s characterize the optimal solution to the problem defined by (10a-f) by the following quadruple

\[\tilde{\Pi}, \tilde{y}, \tilde{x}, \tilde{z}\]

In this scheme, the bank management calculates the optimal strategy and assigns targets as well as allocates bank’s resources among branches. Unfortunately, despite its elegance and simplicity, the centralized formulation of the problem in (10a-f) has little practical value, since it requires bank management to possess unrealistic information transmission and processing capabilities.

First of all such a formulation assumes that the bank management can collect all the necessary information concerning the environmental characteristics of bank’s branches. Notably, from (10c) and the fact that

\[\gamma_i(x_i, z_i, \tilde{x}) \in \Lambda_i \text{ for } \forall i \in \Phi\]

the above assumption implies that the head office have sufficient information concerning the production set of all branches, which is highly unlikely. On the other hand in order to assign targets concerning banking products for each branch, the management should also have sufficient information concerning the local demand and supply for such products.

Suppose that supply and demand of each banking product is determined by the following simple model:
(13) \[
D_j = D_j(p_y, \Psi_y) \quad S_j = S_j(p_y, \Xi_j) \quad \Rightarrow \quad p_y = p_y(\Psi_y, \Xi_j) \quad \forall i \in \Omega \quad \text{and} \quad \forall j \in J,
\]

where \( \{\Psi_y, \Xi_j\}_{i,j} \) are the relevant characteristics that shape the demand and supply function of banking product \( j \) for branch \( i \). This assumption requires the bank management to collect and process information concerning these characteristics which is another component of the local environment, for each branch.

(14) \[
\left\{\{\Psi_y, \Xi_j\}_{i,j}\right\}_{i=j}
\]

Again it is a highly costly procedure to be feasible.

Now, suppose bank tries to decentralize its decision making process by allowing each branch to maximize its own profit. Since the head office is the sole supplier of the collectively used services, branches need to know the availability as well as the cost of such services. For the sake of simplicity suppose that the head office is not a profit-making unit. Therefore it distributes its costs to all branches by charging fees to the services it offers, i.e.

(15) \[
\rho' z = w' y,
\]

where

(16) \[
\rho' = \left[ \sum_{i \in \Omega} \rho_i' \right]
\]

is the vector of “Samuelson (–Oakland) prices for pure public goods used as intermediate goods”\(^{10}\).

On the other hand, following the general practice it can be assumed that bank branches are not allowed to buy or non financial inputs from external sources. Therefore bank branches are constrained by their initial endowment.

Under these assumptions \( i^{th} \) branch’s profit maximization problem can be written as:

(17a) \[
\max \Pi_i = \rho' y_i - w' x_i - \rho' z
\]

\(^{10}\) On the problem of pricing pure public goods, see for example, Jha [10, p. 89–113]. In practice, banks do not use such sophisticated pricing schemes and in general apply a fixed coefficient to distribute the costs of the head office. However, this practice is far from being satisfactory and, in general, raises complaints at branches concerning their burden due to “unnecessary expenses” of the head office.
subject to

\[(17b) \quad \gamma_i \left( y_i, z_i, x_i \right) \leq 0 \]
\[(17c) \quad x_i \leq \bar{x}_i \]
\[(17d) \quad z \leq \bar{z}, \]

where \( \{p_i, w_i, \rho_i\} \) is given.

Obviously this "decentralized" solution still depends on bank management's decision, since branches require collectively used inputs produced by the head office, to produce their banking products. This point is the main difference between "a bank with n branches" and "a system of n banks each with a single branch".

Let

\[(18) \quad \left( \hat{\Pi}, \hat{y}, \hat{x}, z \right) \]

be the optimal solution for branch \( I \), given \( z \). Then, the outcome for the bank, can be expressed by

\[(19) \quad \left( \hat{\Pi}, \hat{y}, \hat{x}, z \right), \]

where

\[(20a) \quad \hat{\Pi} = \sum_{i \in \Omega} \hat{\Pi}_i \]
\[(20b) \quad \hat{y} = \sum_{j \in \Omega} \hat{y}_j \]
\[(20c) \quad \hat{x} = \sum_{i \in \Phi} \hat{x}_i, \]

Obviously one of the tasks of the head office is to choose an optimal level of \( z \) to achieve its objectives.

From these exercises one can derive the following three conclusions:

- The informational requirement of getting a centralized solution, i.e. (11), is prohibitively high, therefore it is, in general, infeasible.
- The approach which leads solution given in (19) can be referred to as "quasi-decentralized" for the following reasons: First of all, the existence of collectively used services effectively prevents full decentralization. Each branch has to take into account the level and the cost of such goods in order to make their profit calculation. Secondly, since branches are not allowed to buy or hire non-financial inputs from out-of-bank sources, they have less autonomy than the term "decentralized" suggests. In fact, the lack of autonomy due to the second reason implies
that the “quasi-decentralized” solution is not Pareto efficient; it can be improved upon simply by allowing branches to trade with their initial endowments (staff, equipment etc)\(^{11}\).

4. Informational Feasibility and Bank’s Profit Maximization Problem

The findings in the preceding section indicate that, a bank management should not overstate its information gathering and processing ability and also shouldn’t undermine the role of the head office’s function as the sole supplier of the collectively used inputs. Therefore a proper strategy for the bank management to handle the profit maximization problem is to try to reduce the informational dimension of the problem.

In practice, a bank management can only observe the following variables (admittedly with some errors of measurement):
- the distribution of personnel and different types non-financial inputs among branches and the head office;
- financial flows among branches;
- outcomes of the operations of bank units.

A glance at this list reveals that although bank management can observe the outcomes of branches, it can hardly be considered as well informed concerning some of the variables that affect them. Particularly, when information gathering and processing costs are taken into account it is practically impossible for the bank management to collect “full” information concerning the “environmental characteristics of branches”.

A rather unsatisfactory solution to this problem is to ignore the differences among bank branches and treat them as “replicas” of each other. If such an assumption is made, then the bank management can solve the optimal resource allocation problem for maximizing bank profit as formulated in (10a–f) for the representative branch, and allocate resources accordingly.

\begin{align}
\text{(21a)} & \quad \max \Pi = \sum_{r \in T} p'_y y_r - \sum_{r \in \Phi} w'_x x_r \\
\text{subject to} & \quad \gamma_0 (z^*, x^0) \leq 0 \\
& \quad \gamma (y, z, x) \leq 0
\end{align}

\(^{11}\) All banks face constraints on the total availability of financial and non-financial resources required for producing financial services. Therefore bank branches can be considered as competing with each other, in order get access to such resources. However, banks through internal rules restrict the scope of competition among branches. For example, branches are not allowed to expand their own clientele at the expense of their fellow branches and personnel allocation, in general, is managed by the head office. On the other hand, access to financial resources is more open to competition among branches.
The apparent weakness of this approach does not prevent it to be used in practice. In many occasions informational constraints led bank managements to ignore the branch specific factors that affect the outcomes of branches. However, as it becomes clear in the case of evaluating the performances of bank branches gains from such a simplification may be much less than the losses due to ignorance\textsuperscript{12).}

On the other hand it is also quite unrealistic to assume that the bank management to be totally uninformed on such issues. In fact, bank managements gather some information through internal communication channels (ex ante) or through internal audit reports (ex post), on branch specific issues. Therefore, a feasible way of approaching this problem is to establish an informationally efficient mechanism to aggregate and transmit the required information.

One way to approach this problem is to assume similar outcome correspondences, i.e. assuming that under same environmental conditions and with the same input configuration, bank branches will respond in a similar way to the information they receive. This assumption can be defended on the ground that, bank operations are well defined and banks pay special attention to train their personnel to satisfy their clients at the same level in all of their branches.

If this assumption is made than a branch’s outcome correspondence (8b) can be written as

\[
\left( y_i, -x_0, -z \right) = \phi\left( \left[ \mu_{ij} \right]_{i \in \Phi, j \in \Lambda} \right), \quad \forall i \in \Omega.
\]

Notice that, in (22), information concerning all variables, except the environmental characteristics of bank branches, can be (and should be) obtained by bank management through in-bank communication, i.e. through accounting and/or statistical data collection activities. However this is hardly comforting, since the costs of gathering and processing information for a large number of branches at a multi-dimensional setting may still be prohibitive.

One way of reducing the informational requirements of bank management is to cluster branches according to their environmental characteristics. A very simple example may be illuminating: Suppose some branches of a bank are in residential areas and the others are in commercial centers. Obviously the pattern of services demanded from these two types of branches will be different. In the former group

\textsuperscript{12) Evaluation of the performances of bank branches is a complex problem and different methodologies ranging from the management accounting approach to the data envelopment analysis (DEA) were proposed to deal with. For the new developments in the use of the DEA in bank branch performance evaluation see, for example, [11, 14, 15, 17] For practical purposes, occasionally an informationally less demanding, therefore less costly, approach than the DEA may suit the purposes of the bank management. This problem is addressed in [3], where multicriterial ranking approach is proposed to deal with the problem of evaluating the performances of bank branches.
of bank branches, deposits and consumer credits will be dominating types of services demanded. In the second group, fund transfers and commercial loans will be the most demanded banking services. In this framework in order to give a target to the branch in question, or make a judgment on its performance, information concerning its group identity along with the statistical information may, indeed, help. Admittedly, such an approach, as any aggregation procedure, introduces a trade-off between precision and simplicity and a bias towards the second. However in practice it has considerable value, since it may help to reduce the probability of making erroneous decision in allocating resources and in evaluating the performances of the staff.

In this case suppose the set of environmental characteristics can be expressed as a finite (reasonably small) disjoint union of its subsets as

\[ \mathcal{I} = \bigcup_{f \in F} \mathcal{I}_f, \quad \mathcal{I}_f \cap \mathcal{I}_g = \emptyset \quad \text{for all } f, g \in F \text{ and } f \neq g. \]

If such a partitioning can be accomplished then (22) can be transformed into

\[ \left\{ y_i, x_i, z_i \right\} = \varphi \left( \left\{ \mu_{i,f} \right\}_{f \in \mathcal{I}_i} \cdot \Lambda_f \right) \quad \forall i \in \Omega \quad \text{and} \quad f \in F. \]

In [6] the above line of reasoning was followed and the following procedure was suggested to cluster bank branches according to their environmental similarities:

- Relevant variables concerning the environment were determined. This was achieved through an iterative method. A proposal was prepared in the Research Department of the Yapi Kredi Bank (a Turkish commercial bank with more than 400 branches) and circulated to a set of staff working both at the head office of the bank and at the branches.
- A measure for each variable was determined. For quantitatively measured variables (such as the population at the vicinity of the branch) their numerical values were used. For qualitative variables (for example the strength of bank competition in the area) scales were introduced.
- The questionnaires were prepared and sent to branches. Branch managers were invited to respond, after taking into account the staff of the branch.
- As a second opinion, area managers of the bank were invited to check the filled forms. They were asked to give their responses separately, and point out mistakes and/or inaccuracies in the responses. These warnings were shared with the branch managers and upon their approval the final data set is obtained.
- Bank branches were clustered according to their environmental characteristics, using the algorithm given in the Appendix.

**Concluding Remarks**

Recognizing the dissimilarities among bank branches, while keeping the extra information gathering and processing costs due to heterogeneity within reasonable bounds, considerably improves the problems that bank managements face in allocat-
ing resources (notably personnel) among branches and in making a fair performance evaluation. Clustering bank branches according to their environmental similarities is one way of solving this problem.

It should also be added that, the process of collecting data from branches also has side benefits. The questions asked to bank branches revealed information concerning each branch’s perception of the boundaries of operation area as well as its capability of information gathering. Based on such information it was possible to identify the zones that are omitted by neighboring branches. That information either led the management to warn the related branches or to open a new branch to address the customers in the omitted zone. On the other hand, the observed deficiencies in bank branches’ capabilities in gathering information concerning the environment they are operating were reported also to the training department to revise the programs to enhance the sensibility of branch staff in following the developments in their vicinity.

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REFERENCES

APPENDIX

Clustering Method

The method used for clustering bank branches can be briefly summarized as follows:

Let $\Omega$ denotes the set of bank branches and denote each bank branch by $i$, where $i \in \Omega$. Suppose that the environmental characteristics of a bank branch is represented by an ordered $w$-tuple $\Lambda = (\delta_1, \ldots, \delta_w)$.

Construct a piecewise linear function which passes through the points $\delta_1, \ldots, \delta_w$ corresponding to the characteristics chosen, i.e.

$$f^\omega : \mathbb{R} \rightarrow \mathbb{R}$$

with

$$f^\omega (r) = \delta_r^\omega \quad r \in \{1, \ldots, w\}.$$

Let $S_\chi = \{V'_1, \ldots, V'_w\}$ be the vector of slopes of the lines that connects such points. Then the problem is defined as clustering the set of vectors, $\{V'_i\}_{i \in \Omega}$. Let’s introduce the following function, $E(V', V'')$, which depends on the distance $d(V', V'')$ between vectors $V'$ and $V''$ in the Euclidean space of vectors $\{V'_i\}_{i \in \Omega}$ in the following way:

13) Adapted, with slight modifications, from [2, p. 67–8].
\[ E(V', V'') = \exp[-k.d(V', V'')] \]

where \( k \) is a constant.

Consider now the clustering \( \{X\} \) on the set of vectors \( \{V^i\} \) such that

\[
(A1) \quad \sum_{t \in T} \lambda \left[ \sum_{V \in \Omega} E(V, V') \right] \rightarrow \max_{\{x\}_{xt}},
\]

where

\[ E(V, X^i) = \sum_{V^i \in X^i} E(V, V^i) \]

and \( \lambda \) is a scaling coefficient. The objective function (A1) obtains higher values if i) the curves inside each cluster are closer to each other, and ii) far from and curves from other clusters, in terms of the function \( E(V, V^i) \).