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On the Social Efficiency in Monopolistic Competition Models[1]

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We consider standard monopolistic competition models in the spirit of Dixit and Stiglitz or Melitz with aggregate consumer's preferences defined by two well-known classes of utility functions – the implicitly defined Kimball utility function and the variable elasticity of substitution utility function. These two classes generalize classical constant elasticity of substitution utility function and overcome its lack of flexibility. It is shown in [Dhingra, Morrow, 2012] that for the monopolistic competition model with aggregate consumer's preferences defined by the variable elasticity of substitution utility function the laissez-faire equilibrium is efficient (i.e. coincides with social welfare state) only for the special case of constant elasticity of substitution utility function. We prove that the constant elasticity of substitution utility function is also the only one which leads to efficient laissez-faire equilibrium in the monopolistic competition model with aggregate consumer's preferences defined by the utility function from the Kimball class. Our main result is following: we find that in both cases a special tax on firms' output may be introduced such that market equilibrium becomes socially efficient. In both cases this tax is calculated up to an arbitrary constant, and some considerations about the «most reasonable» value of this constant are presented.

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1. Introduction

It is a common knowledge that monopolistic competition usually leads to the inefficiency, i.e. market equilibrium differs from the social welfare state. This fact is in line with economic intuition – using their monopoly power firms charge higher prices than the ones which lead to efficient equilibrium. But one special case of efficiency in monopolistic competition model is well known – it is the case when the aggregate consumer's preferences are defined by the constant elasticity of substitution (CES) utility function introduced in [Dixit, Stiglitz, 1977]. This fact makes us think that efficiency in monopolistic competition model is rare but possible thing, so we find it meaningful to take a closer look at this subject. We consider two well-known generalizations of the CES function – the first one is the class of implicitly defined utility functions introduced in [Kimball, 1995] and the second one is the variable elasticity of substitution (VES) utility function introduced in [Zhelobodko et al., 2012]. It was proven in [Dhingra, Morrow, 2012] that CES is the only function in the class of variable elasticity of substitution utility functions which leads to the efficient equilibrium, and we establish similar result for the Kimball class of utility functions. These results look disappointing, but we find a way to «fix» the inefficiency. More precisely, our main result is that for any utility function from the VES or Kimball classes it is possible to introduce special tax on firms such that market equilibrium becomes efficient. One can say that this idea is in the spirit of the Second Welfare Theorem, but on the side of firms, not consumers.

Kimball utility function was proposed in the article [Kimball, 1995], which became one of cornerstones in both Real Business Cycle and New Keynesian literature. Kimball's paper presents «Neomonetarist» model combining standard Real Business Cycle principles and sticky prices a la [Calvo, 1983]. This approach, although not entirely new, stimulated a great deal of research, which resulted in a number of more realistic and accurate models with helpful insights in the certain aspects of economy. Examples include New Neoclassical Synthesis model of US economy in [Smets, Wouters, 2007] and its modification with learning mechanism instead of rational expectations in [Slobodyan, Wouters, 2012], dynamic general-equilibrium model of US and Canadian economies aimed to explain exchange rate dynamics in [Bouakez, 2005], DSGE model with incomplete exchange rate pass-through to trade prices in [Gust et al., 2009], New Keynesian model with labor market frictions in [Riggi, Tancioni, 2010; Sala et al., 2008], model of unemployment in [Givens, 2011], the model of large devaluations in open economy in [Burstein et al., 2007], monetary business cycle model with investment gestation lags and habits in consumption in [Edge et al., 2007], model of endogenous currency choice in [Gopinath et al., 2010], DSGE model with Taylor-type contracting in goods and labor markets in [de Walque et al., 2006], New Keynesian model of inflation dynamics in [Sbordone, 2007]. Kimball aggregator is also used in the model of new economic geography in [Barde, 2008] and monopolistic competition model with cost price change in [Klenow, Willis, 2007].

Our point of interest in this paper is the utility function introduced by Kimball. Its basic principle is flexibility – it is defined via an arbitrary function and may generate any form of demand curve each firm faces. For example it seems plausible from the economic point of view to assume that firms face price elasticity of demand which in increasing in firm's relative price, so it is easier for firm to lose customers after increasing the price than to gain them after decreasing it. Another notable feature of Kimball aggregator is that it is homogeneous of degree one and hence allows to introduce a price aggregator the same way as for constant elasticity of substitution utility function. It is also worth noting that the CES utility function is a special case of Kimball aggregator.

Another generalization of CES utility function is a variable elasticity of substitution utility function introduced in [Zhelobodko et al., 2012] and is motivated by the same idea of overcoming the lack of flexibility of CES utility function, most importantly the independence of firms' price and markups from market size and the independence of firms' size from the number of consumers. Utility functions from the VES class were analyzed extensively in [Dhingra, Morrow, 2012]. In the next section we introduce a standard monopolistic competition models a la [Dixit, Stiglitz, 1977] or [Melitz, 2003] with Kimball and VES utility functions and derive the formulae for taxes which make market equilibria efficient.

2. Models

2.1. Kimball Utility Function

Consider the economy with aggregate consumer and the set of the measure n of monopolistically competitive firms. Denote the level of production of *i*-th firm by x_i. Consumer's utility function Y, which is to be maximized, is defined by the following relation:

$$\int_0^n G(y_i)di = 1, y_i = \frac{x_i}{Y},$$

where $G(1) = 1,\ G'(\xi) > 0,\ G''(\xi) < 0$ for any $x_i \geq 0$.

Denote the total size of the labor force in the economy by L and the common wage by w. Each firm faces variable costs αx_i, fixed costs f, both measured in the units of labor, and also pays taxes $wT(y_i)$. Note that T may be defined as the function of x_i, but the formulae will be slightly more complicated. Since all firms face the same levels of costs, x_i and y_i are also the same for all firms (it could be easily shown formally), so we will use x and y without subscripts where it doesn't create confusion. The goal of consumer is to maximize her utility, the goal of firm is to maximize its profit and the role of government is to collect taxes and then pay them to consumer as transfers.

The problem of the social planner, who solves the utility maximization problem under technological constraints (balance of labor and the structure of utility function), has the following form:

$$\begin{cases} Y \to \max_{y,n}, \\ (\alpha y Y + f)n = L, \\ nG(y) = 1. \end{cases}$$

Expressing Y from the first condition and n from the second one, we get the following function which is to be maximized with respect to y:

$$\frac{LG(y)-f}{\alpha y}.$$

Hence the first order condition is

$$G(y)-yG'(y)=\frac{f}{L}. \tag{1}$$

Denote the solution of this equation by y_w^1. The optimal number of firms may then be calculated as $n_1^w=\dfrac{1}{G\left(y_1^w\right)}$.

Now consider the market equilibrium, which is characterized by the maximization of consumer's utility function, maximization of firms' profits and the financial and labor balances. Consumer solves the problem of minimization of costs of her consumption basket:

$$\begin{cases} Y\int\limits_0^n p_i y_i di \to \min_{p_i} \\ \int\limits_0^n G(y_i)di=1. \end{cases}$$

Forming a Lagrange functional and calculating its partial derivative with respect to p_i, we get (ignoring subscriptions)

$$p=\frac{\lambda}{Y}G'(y), \tag{2}$$

where λ is a Lagrange multiplier. Firm's profit is

$$w\pi = px-(\alpha x+f)w-wT(y)=\lambda yG'(y)-\alpha wyY-fw-wT(y).$$

Maximizing π with respect to y and assuming that the number of firms is high enough so the individual firm can't influence λ, we get the following condition:

$$\frac{\lambda}{w}\left(yG''(y)+G'(y)\right)-\alpha Y-T'(y)=0. \tag{3}$$

As in [Dixit, Stiglitz, 1977; Melitz, 2003] we demand firm's profit to be zero:

$$\lambda yG'(y)-\alpha yY-f-wT(y)=0. \tag{4}$$

Consumer's income consists of wage, firms' profit (which equals to zero) and taxes and is totally spent on consumption:

(5) $$npyY = wL + nwT(y).$$

Together with relation $nG(y) = 1$, equations (2)–(5) form the system of equations which define optimal consumption y_m^1, price and the number of firms n_m^1. Eliminating p, Y, λ from the system of equations (2)–(5) we get the following equation on y:

(6) $$\frac{1}{L} yG'(y)T'(y) - \frac{1}{L}\left(yG'(y)\right)' T(y) = yG''(y)G(y) + \frac{f}{L}G'(y).$$

As we can see, for the given function G values of y in (1) and (6) are the functions of one parameter f/L. We want to find the mechanism supporting the social efficiency of market equilibrium for an arbitrary value of this parameter. Hence, f/L can be expressed through the equilibrium value of y and the efficiency conditions will be satisfied identically. This leads to the differential equation

(7) $$\frac{1}{L} yG'(y)T'(y) - \frac{1}{L}\left(yG'(y)\right)' T(y) = yG''(y)G(y) + G(y)G'(y) - yG'(y)^2.$$

Solve it as a differential equation on T:

$$\frac{yG'(y)T'(y) - \left(yG'(y)\right)' T(y)}{L\left(\left(yG'(y)\right)'\right)^2} = \frac{yG''(y)G(y) + G(y)G'(y) - yG'(y)^2}{\left(\left(yG'(y)\right)'\right)^2}.$$

$$\left(\frac{T(y)}{LyG'(y)}\right)' = \frac{G(y)}{y^2G'(y)} + \frac{G(y)G''(y)}{yG'^2(y)} - \frac{1}{y}.$$

$$\frac{T(y)}{LyG'(y)} = -\int_0^y \left(G(z)\left(\frac{1}{zG'(z)}\right)' - \frac{1}{z} \right) dz + C = -\frac{G(y)}{yG'(y)} + C.$$

Hence,

(8) $$T(y) = \left(CyG'(y) - G(y)\right)L$$

defines the tax rate which makes the market equilibrium efficient for any C. Note that $T(y)$ may be negative and thus be a subsidy. It is also worth noting that if we demand $T(y)$ to be zero, we get a differential equation on $G(y)$, and its solution is $G(y) = ay^{\rho}$ which corresponds to the case of the CES utility function. This proves that the only utility function in the Kimball class such that the market equilibrium is efficient without taxes is the CES function.

Every C leads to the efficient equilibrium, but perhaps it is possible to find the «most reasonable» C. We provide some guesses about this subject. First, for the case of the CES function $G(y) = ay^{\rho}$, there is no need for taxes, so we can assume $T(y) = 0$ and $C = 1/a = 1/G'(1)$. Second, in the case of monopoly it is reasonable to assume that there is also no need for taxes since there is no need for money redistribution. In this case $0 = T(1) = L(CG'(1) - 1)$ and again $C = 1/G'(1)$. These considerations make us think that this value of C is, in some sense, more natural than the others.

2.2. VES Utility Function

In this subsection we derive the similar formula of tax for the class of VES utility functions:

$$U = \int_0^n u(x_i)di,$$

where u is thrice continuously differentiable, strictly increasing and strictly concave on $(0, \infty)$.

Using the fact that outputs of all firms are the same, $x_i = x$, and the obvious fact that creating non-producing firms leads to the losses of social welfare, the problem of the social planner is

$$\begin{cases} nu(x) \to \max_{x,n}, \\ (\alpha x + f)n = L. \end{cases}$$

As we can see, if $u(x) < 0$, it is optimal for the social planner not to produce at all: $n = 0$. As it will become clear soon, in this case the market equilibrium is always inefficient. Hence, adding a constant to u changes the social welfare state, but there is no contradiction with the usual utility theory – the same will happen in the CES or Cobb – Douglas cases. On the other hand, one may add an arbitrary constant to U with no impact on social welfare state and market equilibrium. To overcome this technical difficulty, we impose a normalization on VES utility function: $u(0) = 0$ as in [Dhingra, Morrow, 2012]. Under this condition the optimal level of x may be found from the following equation:

$$u'(x)(\alpha x + f) - \alpha u(x) = 0. \tag{9}$$

The concavity of u guarantees the existence of solution of (9). Denote its solution by x_2^w, and the optimal number of firms is then $n_2^w = \dfrac{L}{\alpha x_2^w + f}$.

In the market equilibrium consumer solves the following problem:

$$\begin{cases} \int_0^n u(x_i)\,di \to \max_{x_i}, \\ \int_0^n p_i x_i di \le wL + \int_0^n w\pi(x_i)\,di + \int_0^n wT(x_i)\,di, \end{cases}$$

and the *i*-th firm solves the profit maximization problem[2]:

(10) $$w\pi_i = p_i x_i - (\alpha x_i + f)w - wT_i(x) \to \max.$$

Lagrange functional for the consumer's utility maximization problem is

$$L = \int_0^n u(x_i)\,di - \lambda\left(wL + \int_0^n wT(x_i)\,di - \int_0^n p_i x_i di \right).$$

We assume that while making her consumption choice, consumer ignores the fact that the income she receives as taxes from firms depend on her choice, so firm's price (ignoring the subscripts) equals to

(11) $$p = \frac{u'(x)}{\lambda}.$$

Substituting (11) into (10), we get

$$\pi = \left(\frac{u'(x)}{\lambda w} - \alpha \right) x - f - T(x).$$

Again, we assume that in equilibrium firms' profit is zero. Hence, the Lagrange multiplier equals

(12) $$\lambda = \frac{xu'(x)}{w(\alpha x + f + T(x))}.$$

Substituting this expression to the firm's profit maximization condition

$$\pi' = \frac{u''(x)}{\lambda w} x + \frac{u'(x)}{\lambda w} - \alpha - T'(x) = 0$$

after some calculations we get the differential equation on $T(x)$:

(13) $$\frac{1}{\alpha} xu'(x)T'(x) - \frac{1}{\alpha}\left(xu'(x)\right)' T(x) = x^2 u''(x) + \frac{f}{\alpha}\left(u'(x) + xu''(x)\right).$$

[2] [Zhelobodko et al., 2012] derives conditions under which first order conditions in this model are sufficient.

Substituting the expression for f/α from (9) to (13), we get

$$\left(\frac{T}{\alpha x u'(x)}\right)' = \frac{u(x)}{x^2 u'^2(x)} + \frac{u(x)u''(x)}{x u'^3(x)} - \frac{1}{x u'(x)}.$$

$$\left(\frac{T}{\alpha x u'(x)}\right) = C - \frac{u(x)}{x u'^2(x)} - \int_0^x \frac{u(z)u''(z)}{x u'^3(z)} dx.$$

Hence,

$$T(x) = \alpha\left[C x u'(x) - \frac{u(x)}{u'(x)} - x u'(x)\int_0^x \frac{u(z)u''(z)}{z u'^3(z)} dz\right]. \tag{14}$$

This choice of tax rate guarantees that firms' output is the same in the social welfare state and in the market equilibrium: $x_2^m = x_2^w$. In order to verify that number of firms is also the same, consider the consumer's budget constraint: $n_2^m px = wL + n_2^m wT$. Substituting (11) and (12), we get $n_2^m = \dfrac{L}{\alpha x_2^m + f} = \dfrac{L}{\alpha x_2^w + f} = n_2^w$. So the number of firms is also the same and hence the market equilibrium is efficient.

To find the «most reasonable» C, again use the intuition that in the CES case there is no need for taxes. Substituting $u(x) = x^a$ in (14), we get $0 = T(x) = Cax^a$, hence $C = 0$.

3. Conclusion

The presented approach allows to «*fix*» inefficient equilibrium in the certain classes of monopolistic competition models. Further research way involve analysis of another classes of utility functions and finding another ways to make market equilibria efficient.

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