

Auctions with Resale: A Survey¹

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Recently emerged, auction theory has become a well-established branch of theoretical economics with important practical applications. As the theory progresses, its basic assumptions become the subject of further investigation and thus new directions emerge. Microeconomics in general and auction theory in particular too often assume away aftermarket interactions, which are a common feature of real markets and have a powerful impact on strategies and incentives. Lately, however, a body of literature emerged that incorporates the possibility of resale into game-theoretic analysis of auctions. This paper reviews this literature. It highlights the role of bargaining power on the aftermarket as one of the main issues in this literature. It then reports how standard auction formats – first and second price auctions in particular – compare in terms of the seller's revenues they generate. Then, this paper shows generalizations of Myerson's approach to constructing optimal auctions when resale is possible; the discussed models require delicate assumptions. Next, the survey covers more specific issues: different approaches to modelling the aftermarket, decisions to enter the primary auction, effects of disclosing information from the primary auction and the role of speculators. Finally, the paper overviews empirical research on auctions with resale, the newest branch of this literature that is developing rapidly.

Key words: auctions with resale; secondary market; aftermarket; resale; auction theory.

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1. Introduction

Nowadays, the auction is a very widespread market mechanism. Auctions are used for finding the best suppliers in public procurement, for selling tickets for sport events, in privatization and for allocating slots in online advertisements. Similarities between these very diverse problems are captured in game-theoretic models of auctions, in which similar agents (called bidders) who want to get some valuable item or items from an auctioneer make strategic bids. The auctioneer herself participates by defining the rules of this auction game, or its format: which bids are accepted and how they get translated into final allocations of items; how payments are determined; what information is released and when, etc.

In practice, the difference between good and bad auction formats can lead to spectacular differences in outcomes. For instance, in 2000–2001 the famous FCC auction brought 22,5 billions of pounds² to British budget, totaling around 100\$ per citizen, while in Switzerland, a comparable country in terms of economic development, it brought only 2,6\$ per citizen. It is argued that this difference is caused by small, but crucial changes in auction design³, such as setting aside a license solely for new entrants.

The standard analysis of auctions assumes away the possibility of resale ignoring the existence of secondary markets⁴ for auctioned items. However, even explicit rules against resale may not be sufficient to suppress it. [Hafalir, Krishna, 2008] present a telling example: while radio spectrum licenses cannot be resold, a Canadian company, TIW, participated in the prime auction via a specially created subsidiary and then successfully sold that subsidiary instead of the license.

In a typical auction model, the set of bidders is fixed exogenously and there are no interactions after the auction is over. Yet, many auctions do not end just in private consumption of the acquired good, but are followed by some further actions. This still can be treated as «consumption» if those actions affect only payoffs and not strategies. However, resale significantly changes bidding decisions so the analysis of the prime auction should reflect the features of the aftermarket.

The goal of this paper is to discuss advances and particular results on auctions with resale, and to highlight through it the main themes of auction theory itself.

There are several excellent books that cover the major issues in the auction theory, such as [Krishna, 2009; Klemperer, 2004]. However, auctions with resale were previously only very briefly covered in section 4,4 of [Krishna, 2009] textbook; also, [Kaplan, Zamir, 2014] describe the main advances in auctions with resale very concisely. There is no book or paper that would contain a detailed survey of the auction models with resale, even though this literature has been growing over the last 20 years. Therefore, this paper fills an important gap and guides an interested reader through this recent literature⁵.

² The data are from [Binmore, Klemperer, 2002]

³ See [Klemperer, 2004; Cramton, Schwartz, 2000] for extensive discussions of theoretical and practical issues of FCC auctions.

⁴ «Aftermarket» and «secondary market» for «good» or «item» will be used interchangeably to label the interactions that follow the «main», «prime» or «primary» auction.

⁵ To keep focus, strategic interactions other than resale are not covered. Market competition between bidders after an auction for licenses, like in [Janssen, Karamychev, 2009] is an example of such interac-

The paper is organized as follows. The second section introduces basic concepts in the auction theory that are most important for later discussion; it can be skipped by readers familiar with auction theory. The third section considers efficiency in auctions with resale; then it delves into comparisons between classic auction formats when resale is taken into account. The fourth section reviews approaches to finding an optimal auction (that maximizes seller's revenue) given resale possibilities. The fifth section covers further theoretical aspects of auctions with resale: the role of bargaining power; endogenous entry, speculation (the role of bidders who have no own use value) and auctions for multiple items. The sixth and final section surveys empirical, mostly experimental studies in auctions with resale.

2. Basic concepts of auction theory

In auction theory the auction is a game. Participants, called bidders, compete for the item(s) put for sale by the auctioneer. The bulk of the literature focuses on single-unit auctions, where only one item is put for sale. The auctioneer typically is not a player in this game, but she may design its rules. When deciding how to bid, participants rely on their use values for the item(s). In the most common set-up with *independent private values* (IPV), the use value of a participant does not depend on values of other participants; values are independently randomly drawn from the same distribution or different ones. In the former case the model is called symmetric, in the latter asymmetric. The IPV model is applicable when participants have different tastes (unknown to competitors and the auctioneer) and are only concerned about the use of the item on sale. Another extreme is the case of *pure common value* auctions, in which there is only one, «true» value and bidders differ because they privately receive different imperfect signals about this value. A classical application of a common value setting is an auction for oilfield drilling rights: companies have different estimates of the oil reserves but the actual reserves and ensuing profits are the same for different participants. A well-known classroom experiment with an auction for a jar of coins is another good example⁶. The literature on auctions with resale predominantly uses the IPV framework. However, the common value setup is sometimes interpreted as a proxy for the presence of future resale opportunities. A realistic model would have both private and common values components: think of buying a painting on an art auction, that gives bidder-specific consumption value and also has an investment element brought about by future resale prospects. However, such models are far less tractable.

Next come the rules of the auction game. Probably, the simplest format is the *first-price sealed-bid auction*: bidders simultaneously submit bids; the bidder with the highest bid becomes the winner; she gets the object and pays the price equal to her bid. This auction is strategically equivalent to the open *Dutch* (or *descending*) auction, in which the announced price gradually decreases from a high value until one bidder declares she is ready to buy the object at the current price; this bidder becomes the winner and pays this price.

tion. The instrumental role of resale in collusion schemes [Garratt, Troger, Zheng, 2009] is also probably more relevant to the issue of means of collusion than to general incentives in the auctions with resale, but deserves a mention.

⁶ Excellent reviews of experimental research on auctions can be found in [Kagel, Roth, 1995; Kagel, Levin, 2014].

Another well-known open auction is called the *English* (or *ascending*) auction: participants outcry their bids, each next bid being larger than the previous ones. The auction continues until no one wants to outbid the current price; the bidder who announced the highest price gets the object at this price. Importantly, all the intermediate bids and the identities of the bidders who made them are publicly known. The fourth important format is the *second-price sealed-bid* auction in which bidders simultaneously submit bids, and the winner is the bidder who submitted the largest bid. However, in contrast to the first-price auction, the winner pays not her own bid, but the second-highest (i.e., the largest losing) bid. In the IPV set-up, both in the second-price sealed-bid auction and in the English auction it is the weakly dominant strategy to submit the bid (or compete up to the price) equal to the true valuation: whatever the other bidders do, a participant cannot gain from misrepresenting her true value. Another important auction format (used, for instance, to model lobbying activity) is the (sealed-bid) *all-pay* auction, in which the highest bidder wins, but all participants pay their bids.

An important issue is the goal of the auctioneer. Two goals typically considered in the literature are maximization of the auctioneer's expected profits and maximization of social welfare. Social welfare is maximized when goods belong to agents who value them most; allocation of goods is called «efficient» in this case. A natural way to achieve an efficient allocation would be to find out the true valuations and to assign the item to the agent with the highest value. The challenge is, of course, to create incentives for truthful revelation.

[Vickrey, 1961] showed that bidding the true value is optimal for a bidder in a second-price sealed-bid auction. Indeed, assume the bidder deviates and submits a bid exceeding her value. Then, this has no effect if her value is higher than the second-highest bid (she wins the auction in both cases and the price is not affected) or the bid she submits is still lower than the highest bid (she loses in both cases). If she gets to win the auction by the overstatement while she was losing when submitting the true value, she only loses from this deviation from the truthful strategy: she wins, but overpays for the item. A similar argument shows that understatement cannot be profitable.

The classical English auction results in an efficient allocation even though the auction ends before the true value is announced (when the last competitor stops bidding). The last bid is lower than the winner's true value, but the order of bids is the same as the order of values, so the resulting allocation is efficient. More generally, with symmetric bidders monotone equilibrium strategies (i.e. the ones in which higher values lead to higher bids) result in an efficient allocation.

Optimality in auction theory is synonymous to maximization of auctioneer's revenues. A common way to increase revenues is to set a reserve price. With a reserve price, only bids above the reserve price are accepted and if there is only one bidder, she pays this reserve price. This prevents too low winning bids at the expense of efficiency (as auctioneer cannot sell the item if bidders' values are positive, but below the reserve price).

The most important result on optimality is called the revenue equivalence theorem (proved independently in [Myerson, 1981; Riley, Samuelson, 1981]). It states that when bidders are risk-neutral and have independent identically distributed private values, all auction formats that have a symmetric equilibrium with bids increasing in valuations yield the same expected revenue to the seller provided the bidder with the lowest valuation gets no surplus. In particular, first-price, second-price, all-pay auctions all bring the same expected profit to the auctioneer. We refer the reader to [Krishna, 2009] for the proof and a detailed discussion. This

is a very powerful result, but it relies on strong assumptions. In particular, the assumption that bidders are symmetric is important: when value distributions are commonly known to be different, revenue equivalence breaks down (see [Maskin, Riley, 2000]). Two questions arise naturally: what auction produces the highest expected revenue and how standard auction formats can be ranked by expected revenue? Usually, first- and second-price auctions are compared as most natural, common and simple formats. Without resale the question of revenue ranking does not have a definite answer for ordinary auctions with asymmetric bidders.

[Myerson, 1981] proves that with asymmetric bidders the optimal auction is generally inefficient. Instead of the actual bidder's valuation, [Myerson, 1981] has shown that the optimal auction assigns the object to the bidder with the highest «virtual valuation» provided it exceeds the seller's own valuation. The virtual valuation⁷ is defined as the actual valuation minus the inverse hazard rate of the bidder's value distribution:

$$w(v) = v - \frac{1 - F(v)}{f(v)}.$$

With asymmetric value distributions the highest virtual valuation need not correspond to the highest «real» valuation even in the regular case (that is, when virtual valuations are increasing functions of true valuations). Therefore, optimal auctions need not be efficient. Consider an example: there are two bidders with values distributed uniformly on [0,1] and [0,2] correspondingly. In an ordinary first-price or second-price auction the first bidder never bids above 1, neither does the second bidder. But the bidder's virtual valuations are

$$w_{(0,1)} = v - \frac{(1-v)}{1} = 2v - 1 \quad \text{and} \quad w_{(0,2)} = v - \frac{(1-v/2)}{1/2} = 2v - 2 = w_{(0,1)} - 1.$$

It may seem that in the optimal auction the second bidder is at a «disadvantage»: if both bidders have the same values and equal bids, the first bidder is sure to win the item since her virtual valuation is higher. However, this optimal design strengthens incentives of the second bidder who is otherwise less-motivated due to an «advantageous» difference in initial distributions. This approach allows extracting the maximal surplus from every bidder, but it requires precise knowledge of the value distributions for every bidder and a complicated mechanism to determine the winner (compared to the «highest bid wins» family of auctions). While one can argue that the first issue is merely a matter of approximation of the optimal results, the second issue is often resolved in favor of suboptimal, but simpler mechanisms. However, optimal mechanisms are still important at least as a benchmark – they provide an upper bound on the revenue that can be potentially achieved.

⁷ Note that when the hazard function is well defined and smooth, such inverse function is also well defined.

3. Efficiency and comparison of standard formats

3.1. Efficiency

With symmetric bidders standard auctions lead to efficient outcomes. This is also true for auctions with resale: when the winner in the prime auction has the highest valuation, there is no scope for resale. However, when bidders' value distributions are asymmetric, efficiency is not guaranteed. [Garratt, Troger, 2006] show the existence of inefficient equilibria in a second-price auction with zero-value bidders (speculators). Nevertheless, there is no general rule applicable to all auction formats and aftermarket models.

The simplest auction model by [Ausubel, Cramton, 1999] studies the choice between designing an efficient auction that maximizes the expected winner's value and an optimal auction that maximizes the expected auctioneer's revenue. Authors assume a perfect resale market and derive a simple rule⁸: when all the information is revealed before the aftermarket, the optimal auction is unambiguously preferable for any auctioneer as the aftermarket will take care of efficiency. [Ausubel, Cramton, 2004] constructed such optimal auction by adding a reserve price to the classic Vickrey second-price auction ([Vickrey, 1961]). This auction will assign the sold items efficiently among the bidders⁹.

These results crucially depend on resale being efficient and perfect, the assumption that is not particularly realistic. For example, [Hafalir, Krishna, 2009] show that in first-price auction addition of resale can, actually, decrease efficiency¹⁰.

3.2. Role of bargaining power and competition

To better understand the role of bargaining power, let us first consider a case with interaction between consumers simpler than an auction – monopoly with resale. Basic theory of price discrimination explicitly forbids resale; otherwise simple discrimination would not be feasible: discriminated customers would avoid the monopolist and buy on the aftermarket. Nevertheless, if resale exists, but is not costless and perfect, some degree of discrimination can still be applied. Consider a monopolistic primary seller of tickets (e.g. a theater or a stadium) and the secondary market for tickets that is not perfect (e.g., it involves search efforts and risks of counterfeit). [Calzolari, Pavan, 2006] show that a stochastic trading rule (a «lottery») is required to maximize the seller's revenue: the monopolist screens consumers buy offering (price, probability of purchase) bundles with price increasing with the probability of purchase. A customer's choice of buying probability is used as a signal on the secondary market¹¹. In practice, tickets often do get cheaper as the event becomes closer. Sometimes additional batches of cheap tickets

⁸ Similar in spirit to the famous Coase theorem ([Coase, 1960]).

⁹ Some inefficiency is created by not selling below the reserve price and keeping the item valued by the bidders.

¹⁰ They also provide revenue comparisons for certain value distributions; for first-price auction results also see [Gupta, Lebrun, 1999].

¹¹ One possible implementation is to disclose the price only with some probability and to guarantee non-disclosure for an additional fee. Another is to refuse to trade with some of customers who ask for cheaper tickets.

may become available. However, waiting for them is risky because their availability is not certain in advance. There is intuition behind the use of a lottery: a customer shows willingness to pay for the ticket not only to the seller, but also to potential secondary buyers. A buyer that chooses a lower probability to acquire the ticket, signals that she attaches a low value for it. Therefore, this signal strengthens her negotiation position against a secondary buyer. By paying more to the monopolist seller, a customer also convinces the secondary buyer that opportunity costs of selling ticket are high.

[Zhang, Wang, 2013] show a similar point in an auction with resale and only two bidders where the use value of one of them is commonly known. It is optimal to sell only through this bidder, excluding the other. The higher the bargaining power of this bidder on the aftermarket is, the higher the revenue of the seller and the theoretical maximum is achieved with full bargaining power going to the aftermarket seller.

[Haile, 2000] identifies two strategic effects of increased competition (and, therefore, the prime auction winner's bargaining power on the aftermarket). The first is the *seller effect*: auction winner may resell to more aftermarket buyers when competition is stronger. The second is the *buyer effect*: the item on the aftermarket becomes more expensive with higher competition (because only one bidder becomes the initial winner and the buyer competes with the rest).

[Lebrun, 2010a] shows that an increase of the prime auction winner's market power on the aftermarket increases the auctioneer's revenue. More precisely, he considers a first-price auction with two bidders, and demonstrates that, when the prime auction winner gets a higher probability of making take-it-or-leave-it offer on the secondary market, it leads to higher seller's expected revenue. This result is in line with the buyer and seller effects of Haile: a higher market power on the aftermarket increases the value of winning the prime auction.

3.3. Standard auction forms with resale: revenue rankings

[Hafalir, Krishna, 2008] show that with two bidders, the first-price auction with resale brings higher expected revenue than the second-price one when the winner makes a take-it-or-leave-it offer to the loser¹². [Lebrun, 2010b], discussed in more detail in Section 5.3, extends this result to the full disclosure case, where both the winning and the losing bid are announced after the primary auction.

[Hafalir, Krishna, 2008] assume that virtual valuations of buyers are regular (increasing). However, later [Cheng, Tan, 2010] have shown that once this assumption is relaxed the ordering may be reversed: the second-price auction may bring higher expected revenue. Their model exploits the bid-equivalence result: an auction with asymmetric independent private values generates the same distribution of bids as a pure common-value auction. In several models of auction with resale both strong and weak participants bid in such a way that the probability to win is the same for each of them, in this sense they become symmetric. This result, called «symmetrization», is natural for an auction with a perfect resale market, but it also can be obtained in models with imperfect markets. It can also be explained through common value analo-

¹² [Hafalir, Krishna, 2008] also consider alternative aftermarket arrangements, including monopsony (a situation with the buyer having all bargaining power on the aftermarket) and show that they lead to the same results provided that no information regarding the winning bid on the primary auction is announced.

gy: both bidders have the same marginal gains of winning, so their strategies are symmetric meaning that they behave as if their valuations were common and not private. Just as the bid-equivalence result, it may fail with more than two bidders.

[Virag, 2013] shows that with more than two bidders symmetrization in first-price auction is incomplete: a stronger bidder is more likely to win, but less so than without resale. [Virag, 2016] comes to the same conclusion in the first-price auction with reserve price and shows the possibility of both rankings of second- and first-price auctions.

4. Optimal auction form when resale is possible

Comparison of classic auction forms leads us to a follow-up question: if some formats are better than others, what is the best auction? Revenue maximization and efficiency are generally incompatible. Moreover, choices required to achieve those goals are sensitive to changes in assumptions, one of which is resale possibility. Resale possibility is not important for the efficiency property in a symmetric auction, as an efficient auction will not disclose information that could lead to profitable resale. When everybody believes that the auction winner has the highest use value (for the efficient allocation this only means correct beliefs) there are no potential gains from resale.

The two main results on optimal mechanisms with resale are described below: Zheng's and Lebrun's auctions. They differ primarily in their approach to the aftermarket. While Lebrun uses additional rules to expand the capabilities of simple auctions, Zheng creates a completely original algorithm.

4.1. Recursive auction with resale

[Zheng, 2002] follows a recursive approach to constructing the optimal auctions with resale. He considers the prime auction and the aftermarket as essentially different stages of the same process, so his auction is built from the end: after the current owner sells the item, the new owner uses the same procedure knowing that the next buyer, in turn, will also employ the same procedure. The selection of this iterative process in the first place is not self-evident, but it is unprofitable to deviate once it is in place.

The main idea of Zheng's auction is to ensure that the probability of selling to a specific bidder is such that after the trade this bidder would choose a resale mechanism relying on the same relative virtual valuations as the initial owner. This can happen only under strict conditions on value distributions, otherwise the existence of an optimal mechanism is not guaranteed. As auction changes the ex-post information distribution, it is generally possible to get cycles of resales or even bubble-like behavior (when every resale makes the next one more likely and happening at higher prices).

How should the optimal auction proceed if the conditions are met? Each stage (that ends by selling the item or ending all trade) proceeds in rounds. In every round the current owner assigns bidder with the highest virtual valuation as the «leader». Then, the current owner sets a reserve price, secretly informs the leader of it (at this point the leader can immediately drop out) and solicits bids from every bidder with virtual valuation higher than that of the owner. Only the leader can win the item, other bidders cannot win in this round, but they can prevent the leader from winning and thus get closer to being future leaders themselves. After getting

the bids, the owner transforms the leader's bid (Zheng derived the required transformation) and compares it to other bids. If the leader is outbid, she leaves the auction and a new leader is assigned among the remaining bidders. Otherwise, the leader wins the item and her bid is announced.

If the leader bids too little, she cannot buy the item on the aftermarket, as she is excluded from the optimal auction in the aftermarket. If she bids too much – she overpays. If a non-leader bids too little, the leader wins and might raise up the reserve price too much. On the other hand, if a non-leader bids too aggressively, it also affects the reserve price and makes it more costly to get the item afterwards. This system of incentives ensures that at every stage the process will resemble the Myerson's auction and produce the same expected revenue.

[Zheng, 2002] puts quite abstract regularity conditions on bidders' value distributions that support optimal equilibrium of the recursive procedure. [Mylovanov, Troger, 2009] refine Zheng's assumptions and generalize his results. They have shown that the result in the two-bidder case relies on only one of the initial assumptions – which the probability of getting the item and virtual valuation should increase together. In the n -bidder case another Zheng's assumption is sufficient alone: the current owner should have the same allocation bias as all the possible remaining bidders. This result allows describing what exactly the regularity condition restricts. Suppose we know the distribution of the last bidder's value. If we also know the supports of values distributions for all the other bidders, we can backtrack the unique set of distributions that allows using Zheng's optimal auction. Only the first player can be drawn from several possible distributions. If any buyer has any other value distribution, iterative auction may not achieve Myerson's surplus and is not necessarily optimal. As Mylovanov and Troger note, this is quite a strong assumption, even though it is less severe than the uniform distribution assumption.

Although Zheng's «detail-free» and «commitment-free» approach results in an auction that is optimal in some settings, this optimality depends entirely on the aftermarket being the same procedure. It is easy to find obstacles to this construct. For instance, commitment opportunities may vary between the bidders. Even pairwise heterogeneity or some constraints (from financial to time constraints) could force the resellers to use different forms of resale, upending the optimality result. Therefore, the generality of Zheng's auction is based on strong assumptions.

4.2. Auction with screening levels

[Lebrun, 2012] follows another approach to constructing the optimal auction, based on a different set of assumptions. He builds a mechanism in two special cases: the two-bidder case and the n -bidder case with all but one bidders being symmetric. Rules are simple: ordinary second-price and English auctions with entry fee and a reserve price¹³. However, in other dimensions his auction is still complex: entry fees are calculated from virtual valuations, reserve prices are personalized (and again, calculated using virtual valuations) and the resulting equilibrium is not necessarily unique.

Lebrun uses two-bidder second-price auction with possible resale and also shows the possibility of optimal English auction in the two bidder case and with $n > 2$ bidders when there are two types of bidders – one strong and others weak. Lebrun assumes nondegeneracy of the value distribution and extended resale monotonicity: density must be nondecreasing when transformed in a particular way.

¹³ The role of reserve price is also covered in [Haile, 2000; Virag, 2016].

The «optimal screening level» is defined by Lebrun as the «smallest value at which he [bidder] may receive the item under the optimal allocation»; it is the inverse of the virtual valuation.

The second price auction in this paper has entrance fees, personally calculated for each potential bidder from distributions, as well as a reserve price and screening levels. If both bidders pay the fee, they enter the second-price auction with a reserve price. While assignment rules are just as in the ordinary version of the auction, information disclosure is minimal – a bidder learns only whether she won and at what price; what the other bid was and even whether there was another participant in the auction is not revealed. Those details are essential to shape aftermarket in an optimal way. In the aftermarket the winner makes a take-it-or-leave-it offer. It is equivalent to the first-price auction with a reserve price and only one bidder.

English auction is similar, but adds an assumption that there are $n - 1$ bidders with the same distribution and n 'th bidder is different from them. Under the assumption that the screening levels are identical (as $1, \dots, n - 1$ bidders face the same sets of opponents) and virtual valuation results in a bias towards bidders $1, \dots, n - 1$ auction and the aftermarket are structured as English auctions, where resale happens only among bidders that were not winners.

Note that entrance fees are necessarily interconnected: the fee for each participant relies on information about another (primarily, her value distribution). The purpose of fees is similar to distribution assumptions in [Zheng, 2002]: they ensure proper composition of potential resellers. Only biasing the bids as in Myerson's optimal auction is not enough, complete exclusion of some bidders is also necessary to achieve optimality.

4.3. Worst-case optimal mechanism

Finally, a working paper by [Carrol, Segal, 2016] takes another definition of revenue-maximization: the auctioneer wants to maximize the worst-case expected revenues when resale procedure is not known. In this case it is possible to construct an auction with truthful equilibrium where resale does not happen even under assumptions most conducive to its possibility (the highest-value bidder knowing other bidders' values in the aftermarket and possessing full bargaining power). Even though this auction requires less restrictions on value distributions and is more robust, comparison with other optimal auction formats should take into account the difference in goals of these auctions – the worst-case expected revenue might be much lower than the maximal expected revenue in cases when Zheng's or Lebrun's assumptions apply.

5. Modelling auction's environment

While ranking of standard auction formats and design of the optimal auction are central to auction theory and studies of auctions with resale, they are sensitive to modelling details: how bidders enter the primary auction, what are their value distributions, how they interact after the primary auction. Papers discussing the effects of those details are reviewed in this section.

5.1. Entry

A typical model of an auction assumes an exogenously fixed number of bidders. In contrast, papers reviewed in this section model costly entry which makes the set of participants

endogenous. Costly entry interacts with resale: entry decision and behavior in the aftermarket become linked because entry decision depends on how competitive the aftermarket is, which is, in turn, affected by the entry to the main auction. Those who refuse to pay the entry fee for the prime auction can enter the aftermarket for free, but those who pay get an advantage on the aftermarket if they win the item. This option value to stay out of the main auction depends on the player's type.

[Xu, Levin, Ye, 2013] assume two levels of entry costs (high and low) that are randomly assigned to players drawn from the same value distribution. They show that resale induces the low-entry-cost type to be more aggressive in the main auction and the high-cost type to be less aggressive and characterize a symmetric equilibrium. Furthermore, they show that the revenue equivalence between standard auctions from the «classical» theory holds in the symmetric equilibrium. Numerical estimates in [Xu, Levin, Ye, 2013] show that resale opportunities increase entry, but have an ambiguous effect on efficiency and revenue.

[Che, Lee, Yang, 2013] characterize equilibrium in the second price auction with sequential entry of two bidders and show that there is a threshold in bargaining power division that separates two opposite effects of resale on entry. If reseller's bargaining power is above the threshold, the resale opportunity causes the bidder that enters first to have a higher (relative to no resale case) incentive to enter, otherwise this incentive is lower. This result is in line with all the other results that show the reseller's bargaining power to influence incentives in the auction, but now extended to sequential entry.

[Celik, Yilankaya, 2016] study costly entry with resale. Resale can be driven by asymmetry in distributions of potential bidders' valuations or by the asymmetry of the equilibrium in the prime auction. The authors study the second case: the distributions are the same (and convex), but the bidders use different entry and bidding strategies. Asymmetry of the equilibrium is driven by the cost of entry and, with resale, also by speculative motives. Celik and Yilankaya show that costly participation by itself may cause those asymmetries, but with resale they will be more pronounced. Moreover, the asymmetric equilibrium they construct brings higher social welfare than any other equilibrium they consider (symmetric or asymmetric in either resale or no resale cases). Beyond efficiency it is hard to say much: revenue comparison is ambiguous. The asymmetric equilibrium is robust to alternative decision timing and relaxing costless resale assumption.

[Che, Klumpp, 2015] compare auctions and sequential mechanisms in a costly entry setting inspired by [Bulow, Klemperer, 2009]. While auctions without resale show higher efficiency but lower revenue than sequential mechanisms, with resale comparisons are more nuanced. Both revenues and efficiency are higher with resale than without. While in auctions this is driven by more entry, in sequential mechanisms this result is due to more aggressive bidding strategies.

5.2. Speculation

Standard auctions are very robust to «speculators», i.e. bidders with zero own value for the item. Such a bidder cannot expect positive revenue from active participating (submitting a positive bid) in any standard auction. One would naturally expect the same to hold in auctions with resale, but there is a body of literature that qualifies this claim.

[Garratt, Troger, 2006] consider adding a speculator with commonly known zero own value to the otherwise symmetric and ordinary auction setting. Without resale, the auction is

not affected by the presence of a speculator, as her dominant strategy is to abstain. However, when the speculator is known to want to resale and to be able to do it, she gains from active bidding. The paper shows that this bidding can result in positive profit for the speculator under certain assumptions, but not in the Dutch or the first-price auction.

[Pagnozzi, 2010] focuses on speculation in multi-unit auctions and the issue of «demand reduction» (demand reduction is discussed in Section 5.4 below). He shows that revenues of the auctioneer can be hurt by speculators who affect incentives to reduce demand more than they affect competitiveness of the primary auction.

An interesting analysis of speculation can be found in [Bose, Deltas, 2002]. They show a somewhat counterintuitive result: sometimes it makes perfect sense to exclude not speculators, but final consumers from the auction, when speculators are also important as middlemen. For this to happen, first, final consumers participating in the auction should be only a small part of whole set of final consumers. Second, most final consumers should be able to get the item only through middlemen (we can also think of them as providers of marketing channels). Then, a final consumer that participates in the auction creates the winner's curse¹⁴: when speculators have to outbid this customer, their bids (and, ultimately, the auctioneer's revenues) are lower.

5.3. Information and disclosure

Information flows change beliefs about other's valuations. Hence, some transaction might be beneficial by itself, but hurt a bidder in the aftermarket as it discloses her value.

Important question regarding information is whether the «linkage principle» can be extended to auctions with resale. The linkage principle (due to [Milgrom, Weber, 1982]) states that the auctioneer has incentive to collect and reveal to the bidders all the free information on the item's worth. The main reason to disclose even bad information is that this information is already accounted for in players' bids, as they are conscious of the winner's curse. However, as [Haile, 2003] points out, the common value component that is created by resale opportunities is different from correlation of private information.

[Mylovanov, Troger, 2008] show that in the general situation of informed principal and private values the principal's type does not affect equilibrium when among mechanisms with full information there is one that is *ex ante* (i.e. before the principal learns her type) optimal. However, it is difficult to specify under which circumstances we can expect this condition to hold. One natural example is when the optimal mechanism relies on a secret reserve price that depends on the principal's private information.

[Hafalir, Krishna, 2008] assume that bids are not disclosed after the auction in order to avoid the ratchet effect¹⁵. [Lebrun, 2010b] shows that there exists the equilibrium that circum-

¹⁴ The «winner's curse» refers to common value auctions, where the winner infers from the fact of winning that she has been the most optimistic among all bidders about the item's value. This is bad news for her. A rational bidder takes this potential future bad news into account when deciding how much to bid, so in equilibrium she does not get disappointed by the fact of winning.

¹⁵ The «ratchet effect» arises when the principal wants to use information about the agent's current performance for designing future incentive schemes, which makes the agent concerned about revealing private information in a dynamic environment under the principal's limited commitment. This was a serious problem in the Soviet planned economy. See: [Weitzman, 1980; Freixas, Guesnerie, Tirole, 1985].

vents this problem. Ratchet effect appears when there are several stages of trading (which is almost always the case in auction with an aftermarket) and information disclosure in the first stage may decrease the later revenue, so information is not fully revealed. It is a common problem in contract theory as it often affects contract efficiency. Hafalir and Krishna supposed that the ratchet effect will decrease the auctioneer's revenue, but Lebrun has shown that, under full bid disclosure, equilibrium with the same revenue still exists. However, in this equilibrium the bids in the auction stage are quite complicated.

Finally, the issue of information revelation is related to more general questions about information structure in mechanism design. For example, a working paper by [Dworczak, 2017] describes the set of revelation policies compatible with truthful mechanisms in a very general sense; he does not assume a specific aftermarket structure. He shows that this set consists of «cutoff policies», i.e. those policies that reveal only that the winner have cleared some threshold, but not by how much and withhold any information on any other bid.

5.4. Multi-unit auctions

Classic auction theory assumes only one item on sale not only for parsimony, but also for tractability reasons: even when items are not complementary and bidders' preferences over bundles are relatively simple¹⁶, their bidding strategies become more complicated than in single-unit auctions. However, splitting multi-unit auction into more tractable simultaneous single-unit auctions is not only hard to implement technically, it is often not a good solution to the multi-unit auction problem. A famous example is collusion in FCC auctions, where bidders used the lowest digits in multi-million bids (mere hundreds of dollars) to signal their interest in particular spectrum bids (see more in [Cramton, Schwartz, 2000]).

There are many possible multi-unit auction formats, but those are usually divided into two types: uniform and discriminatory. The difference is whether winners pay their respective bids or all pay the same price. Internet search advertisement auctions are example of the former and Treasury auctions – of the latter. Treasury auctions are run regularly, provide important budget source for US government and are instrumental in pricing securities that is crucial for financial industry.

Financial securities in general are often resold, but Treasury bills' pricing depends on the aftermarket even more than that of an average security. There is a body of research on drawbacks and benefits of different auction types for this specific market (nicely summarized in [Bikhchandani, Huang, 1993]).

Even in the simplest discriminatory auction with unit demand and symmetric bidders, when resale is possible it happens with a positive probability in equilibrium. Moreover, with more than two bidders the equilibrium may not be symmetric and monotone (a counterexample is provided in [Hafalir, Kurnaz, 2015]) if the seller has all the bargaining power and chooses the mechanism.

Pagnozzi studies multi-unit auctions with resale with a focus on the «demand reduction» effect: strong competitor might prefer to buy some units on the aftermarket rather than compete harder in the prime auction. [Pagnozzi, 2007] shows how seller's revenue can increase when

¹⁶ The simplest assumption is unit demand – a bidder wants only one item, but does not care which one of several on sale.

weak bidders are attracted by the opportunity to resell to strong bidder in the aftermarket. However, [Pagnozzi, 2009] shows the possibility of an opposite scenario: speculators may be unwelcome because their entry strengthens the incentives for the strong bidder to reduce demand. [Pagnozzi, 2010] suggests a way to avert demand reduction: to bundle units together so that resale is possible only from the strong to the weak bidder.

6. Empirical evidence

Theory of auctions with resale dates back at most to late 1990s, but empirical research is even younger, mostly published in 2010s with the only exception of [Haile, 2001]. That paper is also not typical as it studies real auctions rather than experimental data. Real auctions have more parameters (and more importantly, unobservables) than experiments, so it is harder to obtain clear identification.

[Haile, 2001] is a direct extension of the author's earlier works. Specifically, it addresses his main theoretical point, the effect of competition on bids and valuations themselves (buyer and seller effects described in Section 3.2).

Timber auctions suit the framework of auctions with resale perfectly: plots with timber are sold long before timber is fit to be harvested, before sawmills get the information about idiosyncratic future demand. Recession in 1980–1981 caused a change in policy, effectively prohibiting resale in US. Not only explicit resale was severely limited, but contract lengths were cut, required deposits and default penalties were raised. Haile's uses the following instrumental variable approach: with resale, each additional competitor means higher valuations; the number of sawmills in a county and its neighborhood can be used to instrument for competition. Number of participants itself could be correlated with unobservable characteristics of the particular auction, biasing the result. He finds that the number of (competing) sawmills significantly increases bids in all specifications before resale is restricted and its effect becomes significantly (almost twice) smaller after the policy change.

[Lang, List, Price, 2011] gather an extensive dataset on timber auctions in British Columbia, Canada; it includes data on about 3000 auctions. Yet, as they cannot measure the level of resale opportunities that creates unobserved heterogeneity, they resorted to laboratory experiment. The experiment consists of two parts: the first-price auction with or without resale and the risk preference elicitation stage. They conclude that risk aversion and resale possibilities create data patterns that can be easily mistaken for collusive behavior.

If risk aversion helps to explain the experimental data, what other ideas could help to accommodate the data? [Georganas, 2011] explores behavioral approaches to the analysis of his experimental data on English auctions with resale. He notes a clear difference between an English auction without resale where people behave in a manner prescribed by the Nash equilibrium, and an English auction with resale, where evidently people play a very different game (even though the same Nash equilibria are still valid). Several behavioral hypotheses are considered; some are shown to be inconsistent with the data. In particular, risk aversion cannot explain all the patterns in the experimental data, although it can be a contributing factor. Such behavioral factors as «joy of winning» and spite are not supported by the data. The author applies two methods for modelling boundedly rational behavior: Quantal Response Equilibrium (QRE) and k -level reasoning and shows that they can explain the data better than ordinary Nash equilibrium analysis.

Recall from Section 3.3 that [Hafalir, Krishna, 2008] predicted symmetrization of bids when resale becomes available: stronger bidders shade their bids while weaker ones speculate. [Georganas, Kagel, 2011] test this prediction and do find this convergence. However, weak types are not as aggressive as they are predicted to be. This could be a sign of risk-averse behavior, but in other cases those bidders are roughly risk-neutral. There is less symmetrization than expected.

Another experimental paper that tests [Hafalir, Krishna, 2008] is [Kosmopoulou, Jog, 2015]. They compare seller- and buyer-advantaged resale regimes and show that seller's advantage, as expected, produces higher bids. Another observation that resale price closely follows the bids, is not so often encountered in this literature and may merit a separate investigation.

[Sara, 2012] looks at behavior in English clock auction and how it is affected by fine-tuning the aftermarket bargaining power. She observes that the more bargaining power the reseller gets, the higher become the bids in the prime auction. Thus, a powerful final buyer causes demand reduction, while a powerless final buyer incites speculatively high bids.

[Jog, 2012] reports two experiments in the first-price auction with resale setting. The first experiment considers various distributions of bargaining power between the seller and the buyer, testing for symmetrization of bids. The theory states that, both in monopoly and monopsony cases, the seller and the buyer should symmetrize their equilibrium bids, which is experimentally demonstrated in the first part of the dissertation. The second experiment is devoted to «double auction» setting with equal bargaining power for both parties. In this case symmetrization is not prevalent.

[Filiz-Ozbay, Lopez-Vargas, Ozbay, 2015] take on an ambitious task – to compare experimentally two multi-unit formats: Vickrey with package bidding and simultaneous second-price auctions. Vickrey auction, presumably, can handle well valuation externalities of packages, while simultaneous second-price auctions have clear computational benefits – they are much simpler to run. It is not a priori clear how resale will change this comparison: the simultaneous auction may achieve higher efficiency and problems with information acquisition in the Vickrey auction can become less severe. Aftermarket is modelled as take-it-or-leave-it offers, as in [Hafalir, Krishna, 2008]). [Filiz-Ozbay, Lopez-Vargas, Ozbay, 2015] compare four cases, two by two: resale versus no resale and Vickrey auction versus simultaneous second price auction. The theoretical prediction is that in both regimes Vickrey auction is efficient, but simultaneous auctions are not efficient with or without resale. However, the experiment shows the highest efficiency in the simultaneous auctions after the resale stage and the Vickrey format with resale demonstrates the lowest final efficiency rates (the relative frequency of efficient outcomes in the data is used as the measure of efficiency). Before the resale stage, simultaneous auctions with resale are the most inefficient (less than half cases are efficient allocations). More importantly, forbidding resale does not improve revenue in this case. While this result is in stark contrast with the theoretical virtues of Vickrey auction, it is in line with the empirical evidence (see [Ausubel, Milgrom, 2006]). The authors note that experimental Vickrey design usually suffers from complexity and subjects do not behave as theory prescribes them to do. «Global» bidders (i.e., players with positive demand for every item on sale) underbid while bidders with positive demand for only a part of the package on sale tend to overbid (which is also in line with «demand reduction» literature). The authors conclude that, to maximize revenue, simultaneous auctions – if possible – should be accompanied by resale possibilities.

7. Conclusion

Theory of auctions with resale is a young and fascinating topic that provides insights into the frontiers of economic theory. Resale opportunities greatly affect many interactions, from ticket-buyers and museums to governments and telecom companies. Yet, due to complexity of the matter, few clear and general conclusions follow from this literature.

One important lesson both from the theoretical and experimental research is that revenues are largely driven by reseller's market power on the aftermarket, be it a speculator as in [Bose, Deltas, 2002] or an ordinary bidder as in [Hafalir, Krishna, 2008] and the literature that stems from it.

Second, a healthy balance between robustness (realism) and theoretical optimality of the solution is paramount for applications. For example, the optimal auction with resale heavily depends on a particular model of aftermarket and comparisons between classic auctions may depend on fine details of value distributions.

Third, linkages between all stages from entry to the prime auction to the final trades on the aftermarket may be as important as the stages themselves. If incentives (or information structure) in the middle of the model are incorrect, even correct understanding of individual stages will bring wrong results.

Finally, the research in the field is far from being complete and merits much more attention.

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